PROBLEMS OF VACUUM ENERGY AND DARK ENERGY

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Abstract

A simple description of the vacuum energy (cosmological constant) problem for non-experts is presented. Basic features of cosmology with non-zero vacuum energy are discussed. The astronomical data which indicate that the universe is filled with an anti-gravitating state of matter are described. The mechanisms which may lead to cancellation of almost infinite vacuum energy down to the astronomically observed value are discussed. The idea of dynamical adjustment is considered in some more detail.

1   Introduction

During the last decade it has been established through different independent pieces of astronomical data that empty space, devoid of the usual matter, is anti-gravitating. It creates gravitational *repulsion* and gives rise to an accelerated cosmological expansion. According to our present-day understanding, this accelerated expansion can be induced either by non-zero vacuum energy, ρ_{vac}, or by some mysterious agent, called dark energy, which has positive energy density ρ_{DE} and negative and large by absolute value pressure, $p_{DE} < -\rho_{DE}/3$. According to the data, the magnitude of ρ_{vac} or ρ_{DE} is quite close to the critical (closure) cosmological energy density, $\rho_c \approx 5 \, \mathrm{kev/cm}^3 \approx 4 \cdot 10^{-47} \, \mathrm{GeV}^4$. Vacuum or dark energy makes approximately 70% of the latter.

One cannot say that this astronomical discovery was a very big surprise for physicists working on quantum field theory since a natural outcome of this theory is a non-zero energy of the ground state, i.e. $\rho_{vac} \neq 0$. However, the value of ρ_{vac}, according to theoretical expectations is $10^{50} - 10^{120}$ times larger than the magnitude allowed by cosmology. Strange, but the commonly accepted philosophy a decade or more ago was the following: if theory predicts something which is almost infinitely big, somehow it must be exactly zero. The established non-vanishing value of ρ_{vac} makes this point of view even more vulnerable.

At the present day we face the following three striking problems which are very important for our understanding of fundamentals of cosmology and field theory:

1. A huge mismatch between theory and observations at the level 100-50 orders of magnitude. Different natural and even experimentally established contributions to vacuum energy lay in this range. What mechanism is responsible for almost complete cancellation of vacuum energy?

2. Why vacuum energy, which must stay constant in the course of cosmological evolution, or dark energy, which should evolve with time quite differently from the normal matter, have similar magnitude just today, all being close to the value of the critical energy density, $\rho_c \sim m_{Pl}^2/t^2$?

3. If universe acceleration is induced by something which is different from vacuum energy, then what kind of field or object creates the observed cosmological behavior? Or could it be a modification of gravitational interactions at cosmologically large distances?

The gravity of the problem of vacuum energy was first emphasized (in modern terms) by Zeldovich [1], who suggested, in particular, that symmetry between bosons and fermions might grossly alleviate it (this work was a cosmological request for supersymmetry three years before the idea of supersymmetry

was put forward [2]). The first attempt to find a mechanism of dynamical adjustment of vacuum energy from a huge magnitude "predicted" by quantum field theory down to ρ_c was made in ref. [3]. In the mid-eighties the puzzle of ρ_{vac} attracted considerable attention and at the beginning of 90th the first two review papers [4] were published. Now, especially after astronomical indications that ρ_{vac} is indeed non-zero, the number of works dedicated to the related problems became explosively large and even the list of new review papers [5] is almost outside the limits of this article.

The paper is organized as follows. In the next section a brief history of lambda-term or cosmological constant (both are other names for vacuum energy) is presented. It is followed (sec. 3) by description of cosmology with a non-zero Λ. In sec. 4 the astronomical evidence in favor of non-vanishing Λ is discussed. Section 5 is dedicated to theoretical estimates of different contributions into ρ_{vac}. In sec. 6 possible ways to resolve huge discrepancy between theoretical expectations and observational data are described.

2   A little history

Cosmological constant was introduced into the gravitational theory in 1918 in Einstein's paper [6], where he tried to cure what he thought was a shortcoming of General Relativity, namely an absence of stationary cosmological solutions. Einstein noticed that equations of General Relativity (GR) would possess a stationary solution in cosmological situation if one added an extra term proportional to the metric tensor $g_{\mu\nu}$ with a *constant* coefficient Λ:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G_N T_{\mu\nu}^{(m)} + g_{\mu\nu}\Lambda, \qquad (1)$$

where $G_N \equiv 1/m_{Pl}^2$ is the Newton gravitational constant. Positive Λ could counterweight gravitational attraction of the usual matter, described by the homogeneous and isotropic energy-momentum tensor $T_{\mu\nu}^{(m)}$ and would allow for a stationary solution. This solution is evidently unstable because Λ must be constant (see below), while the energy density of matter decreases if the universe expands and rises if it contracts.

After the Hubble's discovery of the cosmological expansion, Einstein emphatically rejected the idea of cosmological constant describing it as the "biggest blunder" of his life. On the other hand, LeMaitre considered an introduction of the lambda-term to the theory as one of the greatest achievements in GR. After the premature birth at 1918 the cosmological constant was several times considered dead (erroneously) but it looks very much alive today. For a long time a great majority of astronomers followed the Einstein's point of view because cosmology did well without lambda-term and the idea of self-gravitating

empty space looked repulsive, but even at this period LeMaitre, De Sitter, and later Eddington were active proponents of cosmological constant. This was considered as a one-parametric freedom of geometrical General Relativity theory. Nowadays, in modern language lambda-term is understood as the energy density of the lowest energy state, i.e. of vacuum.

At the beginning of sixties astronomical observations indicated that recently discovered quasars are accumulated near redshift $z = 2$. To explain the observed enhanced quasar population near $z = 2$ it was suggested that cosmological constant was non-zero and the effect of gravity of the usual matter was compensated by anti-gravity of Λ and hence the expansion slowed down near $z = 2$. Many physicists were sceptical about this explanation which happened to be simply the effect of low statistics and at that time George Gamow wrote [7] "lambda rises its nasty head again". After that period for a long time, till the end of nineties a majority of astronomers, as well as particle physicists believed that cosmological constant is identically zero. However, at the end of the previous century several different pieces of data have been accumulated which made it very difficult (if possible at all) to describe cosmological evolution without lambda-term. These data are discussed in sec. 4 but first we briefly describe some features of cosmology with $\Lambda \neq 0$.

3   Cosmology with non-zero lambda-term

Equations of motion (1) implies that $\Lambda = const$ in frameworks of metric theory of gravity. Indeed, if one applies the covariant derivative, D_μ, to both sides of this equation the r.h.s. vanishes because of covariant conservation of the energy-momentum tensor of matter,

$$D_\mu T^{(m)}_{\mu\nu} = 0. \tag{2}$$

The energy momentum-tensor is defined as the functional derivative of the matter part of action, $T^{(0)}_{\mu\nu} = \delta S^{(m)}/\delta g_{\mu\nu}$, and its conservation follows from the condition that the action is scalar with respect to general coordinate transformation.

The Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - (1/2)g_{\mu\nu}R$ is automatically transverse, $D_\mu G^\mu_\nu \equiv 0$ and thus $D_\mu(\Lambda g^\mu_\nu) = 0$. From this condition follows that

$$\partial\Lambda/\partial x^\mu = 0 \tag{3}$$

because, by construction, covariant derivative of metric tensor identically vanishes, $D_\mu g_{\alpha\beta} = 0$. In some works coordinate dependent lambda-term was discussed. As is seen from these simple arguments such a modification of the theory is not innocent and will not be considered here.

According to the modern point of view, cosmological constant is equivalent to the energy-momentum tensor of vacuum, $T_{\mu\nu}^{(vac)} = g_{\mu\nu}\rho_{vac}$ and $\Lambda = 8\pi G_N \rho_{vac}$. Correspondingly equations (1) can be rewritten as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N \left(T_{\mu\nu}^{(m)} + \rho_{vac} g_{\mu\nu} \right) \tag{4}$$

In homogeneous and isotropic case the energy-momentum tensor has the diagonal form

$$T_\mu^\nu = \mathrm{diag}\left(\rho, -p, -p, -p\right), \tag{5}$$

where ρ and p are respectively energy and pressure densities. For the vacuum one $T_\mu^\nu \sim g_\mu^\nu = \delta_\mu^\nu$ and

$$T_\mu^{\nu\,(vac)} = \mathrm{diag}\left(\rho, \rho, \rho, \rho\right), \tag{6}$$

Thus vacuum pressure density is negative and large; by absolute value it is equal to vacuum energy density,

$$p_{vac} = -\rho_{vac} \tag{7}$$

From this equation immediately follows very interesting and important property that energy density of vacuum (or vacuum-like state - see below) remains constant in the course of cosmological expansion. Indeed, the law of covariant energy conservation (2) in homogeneous and isotropic space has the form:

$$\dot{\rho} = -3H\left(\rho + p\right) \tag{8}$$

Hence for $\rho + p = 0$, as is in vacuum case, $\dot{\rho}_{vac} = 0$, this is a special case of eq. (3) Due to this remarkable property the whole huge present-day universe could be created from a macroscopically small piece of (vacuum-like) matter during an early inflationary stage.

According to one of the the Friedman equations, the Hubble parameter $H = \dot{a}/a$ is expressed through the energy density (in spatially flat universe) as

$$H = \left(\frac{8\pi\rho G_N}{3} \right)^{1/2}. \tag{9}$$

Thus $H = const$ and the expansion is exponential, $a(t) \sim \exp(Ht)$. In the case when the cosmological expansion is determined by the usual matter, with $\rho \sim 1/t^2$ then $H \sim 1/t$ and the scale factor rises as a power of time; $a(t) \sim t^{1/2}$ for relativistic matter and $a(t) \sim t^{2/3}$ for non-relativistic matter.

Another Friedman equation expresses acceleration of the cosmological expansion through ρ and p:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}\,(\rho + 3p) \tag{10}$$

For the usual matter, either non-relativistic with $p \approx 0$ or for relativistic with $p = \rho/3$, the acceleration is negative, as one should expect. Normal matter creates gravitational attraction and slows down cosmological expansion. For vacuum the situation is opposite: positive vacuum energy *anti-gravitates*, i.e. creates gravitational repulsion. The same effect could be created by matter with negative energy density but such objects are possibly too exotic to exist. In particular, finite objects with negative ρ would have negative mass and they not only would create gravitational repulsion but would move in direction opposite to the direction of the applied force. Though vacuum state with positive ρ_{vac} is anti-gravitating, it is impossible to create an anti-gravitating object of finite size even if inside it is filled with vacuum(-like) energy. It can be proved that in the frameworks of normal General Relativity the mass of any finite object is given by an integral from the energy density and if the latter is positive, the total mass is also positive. To prove this statement one has to rely on the Gauss theorem and integration by parts which is impossible for infinite systems.

One more remark is in order here. In usual cosmology with vanishing lambda-term the geometry and ultimate fate of the universe has one-to-one correspondence: open (and flat) universe will expand forever, while closed universe will stop expanding and will re-collapse to a hot and dense stage. This follows from the first Friedman equation with non-zero curvature term (compare to eq. (9) for flat universe):

$$H^2 = \frac{8\pi\rho_{tot}}{m_{Pl}^2} + \frac{k}{a^2} \tag{11}$$

where the constant k determines the curvature of the universe; for $k > 0$ the universe is open and for $k < 0$ it is closed. The energy density of the usual matter decreases in the course of expansion as $1/a^3$ for relativistic matter and $1/a^4$ for non-relativistic matter. Hence after sufficiently large time the second term in the r.h.s. of eq. (11) would dominate and if $k < 0$ the Hubble parameter should become zero (expansion stops) and change sign. If the energy density decreases slower then $1/a^2$ (in vacuum case $\rho_{vac} = const$), then the curvature term may be always negligible and expansion would last forever with any sign of k.

The quantity ρ_{tot} which enters eq. (11) is the sum of all contributions to cosmological energy density, $\rho_{tot} = \rho_m + \rho_{vac} + \rho_{rel} + ...$ Their relative fraction is given by the dimensionless ratio

$$\Omega_a = \rho_a/\rho_c, \tag{12}$$

where $\rho_c = 3H^2 m_{Pl}^2/8\pi$ is the critical energy density. It is evident that if $k = 0$, that is the universe is geometrically flat, i.e. $\Omega_{tot} = 1$.

Strictly speaking it is not established that the new form of matter/energy observed by astronomers is indeed vacuum energy. This new contribution to cosmological energy density got the name dark energy. By assumption its equation of state can be written in the same way as equation of state of other forms of matter/energy:

$$p_{DE} = w\rho_{DE} \tag{13}$$

As we have mentioned above $w = 0$ for non-relativistic matter, $w = 1/3$ for relativistic matter, and $w = -1$ for vacuum, see eq. (7). For an accelerated cosmological expansion one needs $w < -1/3$, as follows from eq. (10).

At this stage is worthwhile to make a remark about possible matter fields which could mimic vacuum energy, but possibly with $w \neq -1$. The simplest example is given by a slowly varying scalar field ϕ. Its energy-momentum tensor is

$$T_{\mu\nu} = 2\phi_\mu\phi_\nu - g_{\mu\nu}\left[\phi_\alpha\phi^\alpha - U(\phi)\right] \tag{14}$$

where $\phi_\mu = \partial\phi/\partial x^\mu$ and $U(\phi)$ is the ϕ-potential. It is easy to see that if derivatives ϕ_μ are small then the energy-momentum tensor is dominated by the potential term $U(\phi)$ and has the vacuum-like form, i.e. it is proportional to $g_{\mu\nu}$. This could be realized if the potential $U(\phi)$ is sufficiently smooth. Another possibility is that $U(\phi)$ has a local minimum with $U(\phi) \neq 0$. In such a case the system might stuck in this minimum (false vacuum) for very long time and the cosmological constant would be non-zero till false vacuum explodes. Depending on the order of the phase transition to real vacuum, the process might be smooth and quiet (second order phase transition) or really explosive.

Parameter w for a homogeneous scalar field in homogeneous cosmological background is equal to

$$w(\phi) = -\frac{2U(\phi) - \dot\phi^2}{2U(\phi) + \dot\phi^2} \tag{15}$$

From this expression one can easily see that $-1 < w < +1$, if $U(\phi) > 0$. In fact this is true for any normal theory. To have $w < -1$, theory should be quite pathological, see refs. [18, 19]. If such a large negative value of w is realized cosmological expansion will end up with crushing singularity. As we see in what follows, the present-day data does not exclude $w < -1$, but most probably the simplest possibility $w = -1$ (or $w > -1$) is realized.

4 Observational data

There are several independent pieces of astronomical data which require non-zero vacuum energy or, similar to it, dark energy.

1. Direct observations of cosmological acceleration.

2. Measurements of the curvature of the universe.

3. Measurements of the the total cosmological density of the usual matter.

4. Theory and observation of large scale structures in the universe.

5. Data on the universe age.

Corresponding astronomical tests are discussed in many standard textbooks on cosmology. For recent reviews one can see e.g. refs. [8]. All relevant observations unanimously require

$$\Omega_m \approx 0.3 \text{ and } \Omega_\lambda \approx 0.7 \tag{16}$$

Here Ω_λ is the cosmological fraction either of vacuum or dark energy. An important feature that strongly amplify the reliability of this result is that it is obtained not only from measurements of the same quantity by different instruments and methods but also from measurements of different and unrelated effects. For example, acceleration of the universe is observed through luminosity of high redshift supernovae, SNIa. Absolutely independent measurements of angular fluctuations of CMBR also demand vacuum-like energy to be non-vanishing and of quite close magnitude. Theory of large scale structure formation strongly supports non-zero Ω_λ too. Moreover, measurements of density fluctuations from different sides: from large scales by CMBR and from small scales by study of matter distribution in the universe (so called large scale structure) demonstrated perfect agreement in coinciding range of wave lengths. This shows that the main features of modern cosmology with non-zero Ω_{lambda} are basically correct.

Below in this section we will briefly discuss some of astronomical data which demands non-zero (and large) Ω_λ and small $\Omega_m < 1$.

4.1 Cosmological acceleration

To measure cosmological acceleration one needs to measure universe expansion rate at large distances or, what is the same, at high redshifts $z \simeq 1$. Of course some deviations from a constant speed expansion do exist at close distance, but they are very small. If there are astronomical objects (standard candles) with known luminosity, L, then their observation would allow to determine the

flux-redshift relation. The measured flux, f, permits to determine the distance to the object, $d_L = (L/4\pi f)^{1/2}$, while redshift (by definition) is determined by the Doppler shift of spectral lines. The distance to the object can be expressed through redshift, the present day value of the Hubble parameter and the law of the evolution of the latter with time. For decelerated expansion the distance would be shorter then for the case of constant speed expansion, while for accelerated expansion the distance would be larger. In the first case the objects would be brighter, while in the second they would be dimmer.

Possibly good standard candles are type Ia supernovae. At least those observed nearby seem to be such. Observations [9, 10] of high red-shift SN Ia show that they are systematically dimmer than would be expected for normal decelerated expansion and thus a possible conclusion could be that the universe expands with acceleration and vacuum energy may be non-vanishing. Though there already existed several other pieces of astronomical data indicating in the same direction, this discovery [9] made the final blow to an old point of view that vacuum energy must be identically zero.

Due to great importance of this result one should be very cautious and try other possible explanations of supernova dimming. Dust present in intergalactic medium would suppress the flux from distant object but simultaneously the usual dust (with the particle size comparable to the light wave length) should shift the color of the light towards red. This effect was not observed.

There could be so called grey dust with particle size much larger than the wave length. Such dust would diminish the flux leaving spectrum intact. However, the recent data [10] show that the supernovae observed at higher redshift, $z \geq 1$ become brighter. It is exactly what should be expected if supernova dimming is created by vacuum or vacuum-like energy. Indeed, as we have seen above, vacuum energy remains constant in the course of cosmological evolution, while the energy density of non-relativistic matter evolves as $(z+1)^3$. If the ratio of vacuum to matter energy densities today is $7/3$, as is found from observations, then at redshifts above $z_{eq} = 0.67$ the difference $2\rho_{vac} - \rho_m$ becomes negative and the expansion would be "normal" decelerated. Thus at $z > z_{eq}$ one would expect that the dimming of supernova with respect to the "constant speed" expansion should gradually decrease and ultimately turn into brightening. The observation of non-monotonic behavior of dimming with redshift makes also unlikely a possible explanation of dimming by SN evolution effects.

Thus it seems that the most natural explanation of the dimming of the high redshift SN Ia is the accelerated expansion of the universe. The data are sensitive, roughly speaking, to the difference of ρ_{vac} and ρ_m and does not allow to determine both but together with other astronomical observations a separate determination of density of the usual (dark) matter and dark energy is possible. In particular, the recent analysis [10] yields $\Omega_\lambda - 1.4\Omega_m = 0.35 \pm 0.14$,

under assumption that the equation of state of dark energy is $w = -1$. The data of ref. [10] are compatible with this assumption giving $w = -1.02^{+0.13}_{-0.19}$. If the universe is flat (see the next subsection) i.e. $\Omega_{tot} = \Omega_m + \Omega_\lambda = 1$, then $\Omega_m = 0.29^{+0.05}_{-0.03}$, independently on the data on the large scale structure (LSS) of the universe.

4.2 Curvature of the universe

The curvature of the universe can be accurately determined from the classical cosmological angular size test applied to angular fluctuations of cosmic microwave background radiation (CMBR) (for a simple review see e.g. ref. [11]). The first (highest) acoustic peak in the angular spectrum of fluctuations corresponds to the sound wave with the length equal to the so called sound horizon d_s at the epoch of hydrogen recombination. The latter differ from the Hubble horizon by the speed of sound factor, $c_s = 1/\sqrt{3}$. If we know d_s and know the angular size at which we observe it today we are able to say what is the geometry of the universe. For an open universe the angle would be bigger that for the flat one, while for the closed universe the angle would be smaller. All recent measurements of angular fluctuations of CMBR show the position of the first acoustic peak exactly at the place which corresponds to the flat universe. According to the most precise data of WMAP [12] combined with other astronomical data the result is:

$$\Omega_{tot} = 1.02 \pm 0.02 \tag{17}$$

Thus in addition to the "difference" of ρ_{vac} and ρ_m their sum is also measured. From these one can find ρ_{vac} and ρ_m separately as is described in the previous subsection. Moreover, the positions and heights of acoustic peaks in CMBR angular spectrum depend upon the law of cosmological expansion and hence upon the fraction of non-relativistic matter. Thus an analysis based on the complete set of WMAP data [12] (and not only on the position of the first peak) allows to determine Ω_m without invoking results of other measurements and gives $\Omega_m = 0.29 \pm 0.07$ in good agreement with direct determination of the latter. If one includes data and theory of LSS formation then the fraction of matter would be $\Omega_m = 0.27 \pm 0.04$ [12].

Simultaneously one can determine the equation of state of dark energy. According to ref. [12] it is $w = -0.98 \pm 0.12$.

4.3 Mass density of usual (dark) matter

There are several ways to determine the cosmological energy density of clustered matter[1]. One possibility is to study the galaxy velocity fields assuming that galactic peculiar motion (with respect to the Hubble flow) is induced by gravitational action of surrounding matter. Different methods and samples of galaxies give consistent results [13] averaging around $\Omega_m = 0.3$.

The study of equilibrium of hot gas in rich clusters permits to measure the ratio of the mass of the baryonic component to the total mass of the cluster. This ratio was found to be quite large, $\Omega_b/\Omega_m \approx 0.15$. Since it is known from BBN and independently from CMBR that $\Omega_b \approx 0.05$, we find again $\Omega_m \approx 0.3$.

The third method of determination of the mass density of matter in the universe is based on determination of cluster abundances at different redshifts z. In the universe with $\Omega_m \sim 1$ cluster formation strongly grows with time and number of clusters today must be much larger than, say, at $z \simeq 1$. On the contrary, in low Ω_m-universe the number of clusters at the present time and around $z = 1$ should be approximately the same. Observations demonstrate very small change in cluster abundances and thus support low mass cosmology. Analysis made in different works [14] lead to the conclusion that $\Omega_m = 0.1-0.4$. Though the dispersion of the results is quite high, they reliably exclude large values of Ω_m.

Matter inhomogeneities along the line of sight to background galaxy distort its image due to gravitational lensing effect. This is a basis of one more method of "weighting" the universe. Results of different measurements are summarized in review [15] and all agree with low Ω_m.

Thus we see that several independent astronomical measurements give the consistent result $\Omega_m \approx 0.3$.

4.4 Universe age

Long existing discrepancy between a relatively large value of the Hubble parameter $H \approx 70$ km/sec/Mps and large universe age, t_U, is nicely resolved if vacuum energy is non-zero. If we compare two regimes of cosmological expansion, accelerated and decelerated, then with the same value of the Hubble parameter at the present time $H = \dot{a}/a$, expansion was slower in the past for accelerated regime. it means that to reach the same magnitude of H more time was necessary and the accelerated universe should be older.

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[1]According to eq. (3), vacuum energy must be uniform, however one should bear in mind that if dark energy is not just simple vacuum energy, but the energy of some new weakly interacting field, then depending upon the properties of this field, dark energy may also be clustered.

The universe age can be expressed through the present day values of the Hubble parameter H_0 and fractions of different forms of energy as:

$$t_u = \frac{1}{H_0} \int_0^1 \frac{dx}{(1 - \Omega_{tot} + \Omega_m/x + \Omega_v x^2)^{1/2}} \qquad (18)$$

where $H_0^{-1} = 9.8 \cdot 10^9 h^{-1}$ years and dimensionless parameter h according to modern data is about 0.7. Hence in flat matter dominated universe with $\Omega_{tot} = \Omega_m = 1$ the universe would be only 9.3 Gyr while nuclear chronology (reviewed in ref. [16]) and the age of old globular clusters (reviewed in ref. [17]) indicate much larger age, $t_u = 12 - 15$ Gyr. For flat universe with $\Omega_m = 0.3$ and $\Omega_v = 0.7$ the universe age according to eq. (18) is $t_u = 13.8$ Gyr in good agreement with the range quoted above.

5   Contributions to vacuum energy

Quantum field theory and particle physics predict that there are several huge contributions into vacuum energy, while we see from observations that all these contributions are miraculously canceled almost to nothing on the scale of particle physics but just of order of unity on the present day cosmological scale.

It is well known from quantum mechanics that the ground state energy of an oscillator is not zero but $E_0 = \omega/2$. Quantum field theory deals with infinitely many oscillators labeled by their wave number $\mathbf{k}$ and the energy of the ground state (vacuum) is infinitely large. For example the energy density of a bosonic field is given by the integral:

$$\langle \mathcal{H}_b \rangle_{vac} = g_s \int \frac{d^3k}{(2\pi)^3} \frac{\omega_k}{2} \langle a_k^\dagger a_k + b_k b_k^\dagger \rangle_{vac} = g_s \int \frac{d^3k}{(2\pi)^3} \omega_k = \infty^4 \qquad (19)$$

where g_s is the number of spin states, $a_k^\dagger$ and a_k are creation-annihilation operators for particles, b_k are the same for antiparticles, and $\omega = \sqrt{k^2 + m^2}$.

Vacuum energy of a fermionic field is given by a similar integral with a sign difference:

$$\langle \mathcal{H}_f \rangle_{vac} = g_s \int \frac{d^3k}{(2\pi)^3} \frac{\omega_k}{2} \langle a_k^\dagger a_k - b_k b_k^\dagger \rangle_{vac} = g_s \int \frac{d^3k}{(2\pi)^3} \omega_k = -\infty^4 \qquad (20)$$

Thus if the theory is symmetric with interchange of bosons and fermions i.e. there exist an equal number of bosonis and fermionic states with equal masses (pairwise) then the energy of vacuum fluctuations would naturally be zero [1].

Later, when supersymmetry was theoretically discovered [2] it was found that indeed $\langle \mathcal{H}_{tot} \rangle_{vac} = 0$ if the symmetry were unbroken. We know, however,

that supersymmetry is broken and supersymmetric partners of the known particles, if they exist, should be much heavier, at least in 100 GeV-TeV range or even above. Moreover, theoretically preferred breaking of supersymmetry, so called soft breaking, demands that vacuum energy is non-zero:

$$\langle \mathcal{H}_{tot} \rangle_{vac} \sim m_{susy}^4 \geq 10^8 \text{ GeV} \geq 10^{54} \rho_c \qquad (21)$$

Hence one should exclude the most appealing theoretically models of supersymmetry breaking but there exists a local generalization of global supersymmetry, namely supergravity which is free from this constraint. Still the natural value of vacuum energy in supergravity is about $m_{Pl}^4 \sim 10^{76}$ GeV and its near vanishing demands fine-tuning of the parameters more then by 120 orders of magnitude.

Theories with spontaneously broken symmetry create an additional problem for nullification of vacuum energy. Effective potential of the scalar (Higgs) field responsible for spontaneous symmetry breaking typically has the following form:

$$U(\Phi) = -m_\Phi^2 \Phi^2 + \lambda \Phi^4 = \lambda(\Phi^2 - \eta^2)^2 + m_\Phi^4/\lambda \qquad (22)$$

In broken symmetry phase $\Phi = \eta$ and the last term in the above equation leads to huge vacuum energy $\rho_{vac} = m_\Phi^4/\lambda$. We have to believe that, by some mysterious reason, this term should be subtracted from these expressions. In other words, vacuum energy in unbroken phase should be non-zero to ensure its vanishing in broken phase.

Thus in the course of the universe expansion and cooling down there existed several phases when vacuum energy was non-zero and large by modern cosmological standards. During inflationary stage the universe was dominated by vacuum-like energy of the inflaton field. Later on, vacuum energy and energy of hot primeval plasma were comparable but still energy density of matter was dominant, except for periods near possible first order phase transitions with sufficiently strong super-cooling. Vacuum-like energy density should be sub-dominant during big bang nucleosynthesis (BBN). Otherwise successful predictions of the latter would be distorted. It was possibly sub-dominant during all the time from BBN to practically present stage and only from redshift $z_{eq} \approx 0.67$ (see sec. 4.1) the dark energy became dominant and remains such now.

The change of vacuum energy at the electro-weak phase transition is about $\Delta \rho_{vac}^{EW} \sim 10^8$ GeV^4, which is already huge in comparison with $\rho_c \sim 10^{-46}$ GeV^4, but at Grand Unification scale it is by far larger, $\Delta \rho_{vac}^{GUT} \sim 10^{64}$ GeV^4. The change of vacuum energy at QCD phase transition is minor with respect to those mentioned above, it is "only" 45 orders of magnitude larger than the cosmological energy density.

Still, though the QCD contribution to vacuum energy is the smallest of all above, it has a very special standing, because in a sense it is experimentally

known quantity. Hadron properties could be explained in the frameworks of QCD only if there are non-vanishing quark [20] and gluon [21] condensates. The properties of these condensates are well established and it is known that their (vacuum) energy if at the level 0.1 GeV^4. Thus we know that vacuum is not empty and the energy density of some particular contributions to vacuum energy from quarks and gluons is about 45 orders of magnitude larger than the observed value.

It is difficult to avoid the conclusion that something "lives" in vacuum which very accurately, but not completely, cancels out the energy density of the QCD condensates and of some other much larger contributions. This something is not related to quarks and gluons through QCD interactions because otherwise it would be observed in direct particle physics experiments. One may say that very heavy fields/particles may escape experimental observations but it is hardly possible that a heavy field may take care of vacuum energy at the level of 10^{-46} GeV^4.

6 Possible ways to solve the problem

As was already mentioned above, there are two different problems:
1) Why vacuum energy is not infinitely, or almost infinitely, large?
2) What creates the observed universe acceleration? Is it a new form of energy or gravitational forces are modified at cosmologically large distances?
In what follows we will concentrate on the first problem because it seems very likely that its solution should give an insight into the physical nature of the dark energy. On the other hand, more observational data on phenomenology of the dark energy may present an important clue to the solution of the first problem.

The simplest, but probably unsatisfactory, suggestion is to say that the sum of all contributions to vacuum energy is canceled out by a **subtraction constant**, ρ_{sub}, which is chosen quite precisely to cancel the present day value of the vacuum energy (why today but not at some earlier stage of cosmological evolution?) with 100 orders of magnitude precision, but not exactly, leaving behind a small remnant $\sim \rho_c$. Though it seems impossible to forbid such a point of view formally, it does not look very attractive.

Another approach to attacking the problem is based on **anthropic considerations**. If Λ is stochastically distributed variable (but why?) then one may estimate with which values of Λ the probability of life is the largest. Since density fluctuations which gave rise to galaxy formation became frozen when vacuum energy started to dominate the probability of life in the worlds with a large Λ would be small [22]. If the probability distribution of Λ is uniform, then one would expect that life is most probable in unverses where vacuum energy started to dominate at the epoch of galaxy formation. It is consistent

with what is observed in our universe. On the other hand, some time ago, prior to the inflationary idea [23], the attempts had been done to invoke anthropic considerations for a solution of the fundamental problems of the Friedman cosmology. Now, instead of the anthropic solution, we have much better and testable cosmological scenario.

A natural suggestion to invoke a **symmetry** which demands almost complete vanishing of vacuum energy did not lead to any progress up to now. Such a symmetry must include bosons and fermions on equal footing because both kind of fields possess non-vanishing energy-momentum tensor coupled to gravity and thus such symmetry should be somehow related to supersymmetry, which is known to be badly broken. Moreover, an exact symmetry in flat spacetime may be broken in curved one. All that makes the symmetry approach very difficult to realize.

Infrared instability of massless fields in De Sitter background [24] may somewhat diminish the original vacuum energy by quantum back reaction. Earlier works on this mechanism [25] were criticized in papers [26] where was argued that the back reaction is too weak to create a noticeable effect. Activity in this field is continuing and it is premature to bury it; for recent development see ref. [27].

Modification of gravitational interaction at large distances is attracting more and more attention, motivated by higher dimensional theories, see e.g. [28] or by doubling the number of gravitons [29]. There are much more papers on the subject and it is impossible even to mention them all in this talk, moreover, they are not directly related to the subject because they do not address the problem of cancellation of the huge contributions into ρ_{vac} which is the main interest here. Older approaches to solve the problem of cosmological constant with modified gravity pursued the possibility that in a modified theory the term in the energy-momentum proportional to $g_{\mu\nu}$ does not gravitate. If such an idea were realized it would solve the problem because in this case the vacuum energy density would be unobservable. However, it seems impossible to achieve that because in the course of a phase transition the equation of state can change from $p = -\rho$ to e.g. $p = \rho/3$ and an absence of gravity in the first phase and its presence in the second one would be incompatible with the demand that massless graviton must interact with a conserved source.

Adjustment mechanism seems most promising to me and because of that a separate subsection is devoted to it.

6.1 Adjustment mechanism

The idea of adjustment mechanism is quite simple: vacuum energy might stimulate formation of a condensate of some field, coupled to curvature of space-time, whose energy density compensates the energy density of the source [3]. In fact

this is a general physical principle which was formulated centuries ago by Le Chatellier. Adjustment mechanism has potential to solve both problems: to compensate vacuum energy down to acceptable value and leave behind a non-compensated remnant of the order of $\rho_c(t)$. This property of adjustment mechanism had been formulated [3] long before universe acceleration was discovered and the problem of vacuum energy attracted common attention. However, a serious shortcoming of adjustment mechanism is that, up to now, none of the models considered in the literature leads to the realistic cosmology. Still, despite that, some general features of adjustment could remain in the future more successful models and thus we will discuss this mechanism here.

In the first attempt [3] to resolve the vacuum energy problem a massless scalar field ϕ non-minimally coupled to gravity was introduced. Its Lagrangian has the form:

$$\mathcal{L}_0 = \partial_\mu \phi \partial_\nu \phi / 2 + \xi R \phi^2 \tag{23}$$

Equation of motion of this field in De Sitter background with constant scalar curvature R is unstable if the constant ξ is positive. It corresponds to tachyonic, negative mass squared, case. As a result, a homogeneous solution of the equation of motion, $\phi = \phi(t)$, starts to rise exponentially while the energy-momentum tensor of ϕ remains negligible with respect to the vacuum one. Later, when $T_{\mu\nu}(\phi)$ becomes comparable to $T_{\mu\nu}^{(vac)}$ its back-reaction would slow down the expansion rate from the exponential to a power law and simultaneously the exponential rise of ϕ turns into $\phi \sim t$. So far so good, but the effective gravitational coupling in this model $G_N = 1/(m_{Pl}^2 + \xi \phi^2)$ is strongly decreasing with time which seems to be not the case in real world.

There have been several other attempts to implement this idea with a scalar field but they all were similarly unsuccessful, for a review and list of references see e.g. the papers [4, 5]. It was argued by S. Weinberg (first paper in ref. [4]) that there is no-go theorem which does not allow to implement adjustment mechanism with a scalar field. As we know, however, many or all no-go theorems in quantum field theory have been successfully over-went and activity in this field still continues.

An interesting suggestion was made recently in ref. [30], where the authors proposed to modify kinetic term of the compensating scalar field as follows, $\mathcal{L}_{kin} \sim (\partial \phi)^{2q} / R^{2m(2q-1)}$. Since in this case the equation of motion of ϕ looks roughly speaking as

$$D^2 \phi + F(R, \phi) = 0 \tag{24}$$

where the function F vanishes at $R = 0$ the solution of this equation has an equilibrium point at $R = 0$. However, the authors of this paper requested that the power m should be sufficiently large, $m > 3/2$, to ensure stability

of the solutions. It can be shown that in this case the universe expansion always remains so fast that the contribution of ordinary matter into cosmological energy density would be negligible. An attempt to overcome this problem was done in ref. [31] where the models with $\mathcal{L}_{kin} = (\partial\phi)^2/R^2$ or even with $\mathcal{L}_{kin} = (\partial\phi)^2/R|R|$ have been considered. It was found that there exist solutions of equations of motion which indeed lead to cancellation of vacuum energy and the cosmological expansion changes from exponential (De Sitter) regime to the Friedman regime which could be very close to that in radiation dominated universe. Some solutions are even stable with respect to small perturbations but still detailed features of such cosmology are far from realistic. Quantization in this approach remains problematic but, first thing first, we need to find a realistic solution to the classical problem, which still remains to be found, and after that may start bothering about quantum effects.

Scalar field is not the only possible candidate for the role of compensating field, higher spin fields, vector [32], or tensor [18] might also do the job. In such models a condensate of time component of vector field, V_t, or tensor field S_{tt} is developed in De Sitter space-time. The energy of this condensate cancels its creator, the vacuum energy. Though Lorenz invariance is broken in such models, possible effects of its breaking are not dangerous. The unstable classical mode which compensates positive vacuum energy appears because of "wrong" sign of the corresponding term in the Lagrangian but fluctuations about this background are well behaved and non-dangerous[2]. Unfortunately the versions of the models with vector and tensor fields do not lead to realistic cosmologies as well. In particular, a special version of tensor field condensation considered in ref. [32] leads to a too strong time variation of the gravitational constant [34].

To summarize, no workable adjustment mechanism is found up to now but one should not be too pessimistic - it is not yet proved that such mechanism cannot exist. At the moment it seems to be the only approach which may dynamically solve the problems of huge and small vacuum/dark energies in one blow.

7   Conclusion

A very important psychological consequence of the discovery of the accelerated cosmological expansion is that it attracted much deserved attention to the vacuum energy problem. However, the bulk of publications on the subject deal only with the "small" part of the problem, namely, what is the origin of the acceleration and neglect the "large" part - what is the mechanism of almost exact cancellation of vacuum energy. It is quite probable that this two

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[2]Similar idea with condensation of time derivative of a scalar field was discussed recently in ref. [33] for possible explanation of cosmic acceleration.

parts "large" and "small" are tightly connected and one cannot be understood without the other.

Some enthusiasm is expressed about a solution of the "large" problem on the basis of higher dimensional theories. However, no noticeable success was achieved on this road and anyhow we live in 4-dimensional world and the problem should be solved there.

As I have already mentioned, my best choice is an adjustment mechanism and, though all attempts in this direction still did not reach the goal, the idea, which follows from adjustment ideology, that there exists a new form of cosmological energy with an unusual equation of state appeared long before before the accelerated expansion was discovered. So it seems that more work on this mechanism is desirable and may even be successful in the nearest future.

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